

# Contrast Enhancement Technique for Flat EEG Image using Interval-Valued Type-II Fuzzy Sets

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## ABSTRACT

This paper presents a novel contrast enhancement technique specifically designed for flat EEG image. It aims to visualize the path of brainstorm that occur during seizure by reducing the spread of electrical potential in the cluster centres. The proposed approach involves the conversion of images into interval-valued fuzzy representations, followed by enhancement using the type-II fuzzy approach. The experimental results reveal that the integration of interval-valued and type-II techniques yields superior results compared to employing the interval-valued technique independently. Performance analysis, such as the structural similarity index measure (SSIM), demonstrates that the proposed method outperforms the non-combination approach.

## 1. Introduction

Image processing is a prominent field of study that involves the application of algorithms to manipulate input images and generate corresponding output images. Image enhancement is a crucial component of image processing that uses a variety of transformation techniques to increase the visual quality of an input image. For instance, one common enhancement technique involves increasing the brightness of a dark image or enhancing the contrast to enhance the visibility of details. Another approach is to detect and emphasize intensity edges within an image, enabling the highlighting of object boundaries or facilitating object detection. Through the utilization of these image enhancement techniques, the visual appearance and interpretability of images can be significantly improved. Fuzzy set theory has emerged as a powerful framework for addressing the inherent uncertainty present in digital image processing. Proposed by Zadeh in 1965 [1], this theory has gained significant traction and has become widely utilized in various domains, with digital image processing being a particularly notable area of application. The versatility and effectiveness of fuzzy

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set theory in handling uncertain and imprecise data within this field have contributed to its widespread adoption. Its integration into digital image processing techniques has yielded substantial advancements in addressing the challenges associated with uncertainty, thereby enhancing the quality and reliability of image processing outcomes. In 1975, Zadeh introduced the concept of type-II fuzzy sets as a significant extension to the existing framework of fuzzy sets. In this advanced framework, the membership function is intrinsically characterized by fuzziness, in contrast to the crisp membership values traditionally associated with fuzzy sets. The introduction of type-II fuzzy sets facilitates a subtler representation of uncertainty by explicitly incorporating the inherent fuzziness of the membership function itself. This development enriches the modelling capabilities of fuzzy set theory, allowing for a more comprehensive treatment of complex and uncertain phenomena.

According to Bustince [2], in practical applications where the precise knowledge of membership functions for constructing fuzzy sets is lacking, it becomes appropriate to represent the membership degrees of individual elements within the fuzzy set using intervals. This conceptualization leads to the development of interval-valued fuzzy sets, an extension of fuzzy sets. Interval-valued fuzzy sets are characterized by assigning membership degrees to elements as closed subintervals within the interval  $[0, 1]$ . Consequently, this framework not only captures the inherent vagueness associated with indeterminate class boundaries but also enables the intuitive handling of uncertainties arising from the lack of information. Furthermore, the study on flat EEG image enhancement was carried out by [3-5]. In [3], the flat EEG image underwent enhancement through the utilization of classical, ordinary fuzzy, and advanced fuzzy approaches. Meanwhile, the process of flat EEG image segmentation was executed through the implementation of a multi-level thresholding technique based on fuzzy entropy as discussed in [4]. A comparative analysis of advanced fuzzy set on Flat EEG image was discussed in [5]. The advanced fuzzy methods involved intuitionistic fuzzy set and type-2 fuzzy set.

The application of fuzzy set on enhancing images by using adjustable intensifier (INT) operator was proposed by [6]. The adjustable INT operator is utilized to decrease membership values in the low region while increasing them in the high region. In [7], a new intensifier operator was introduced to enhance fuzzy images. Fuzzy sets constitute a mathematical framework that accommodates ambiguity and imprecision in data representation. By allowing membership degrees to assume values between 0 and 1, fuzzy sets enable the modelling of partial memberships, thus facilitating more flexible and nuanced data analysis. type-II fuzzy sets extend the conventional fuzzy set theory by considering the fuzziness of the membership function itself. This allows for the incorporation of higher levels of uncertainty and ambiguity, resulting in a more intricate representation and analysis of complex phenomena. Interval-valued fuzzy sets represent an extension of fuzzy sets where the membership degree of each element is expressed as a closed subinterval within the range  $[0, 1]$ . This framework provides a flexible means to represent uncertainty, vagueness, and imprecision in data, enabling a comprehensive treatment of diverse real-world phenomena.

## **2. Preliminaries**

This section provides a concise overview of the fundamental concepts underlying flat EEG, type-II fuzzy sets, and interval-valued fuzzy sets.

### *2.1 Flat EEG*

In 2008, Zakaria [8] has designed a technique known as flat EEG involves the transformation of high-dimensional EEG signals into a lower-dimensional representation, thereby enabling their

visualization on a Cartesian plane (see Figure 1). The term "flat EEG" pertains to a specific electroencephalogram (EEG) recording characterized by a consistent and uniform signal amplitude. Understanding the concept of flat EEG is essential for comprehending the analysis and interpretation of EEG data.

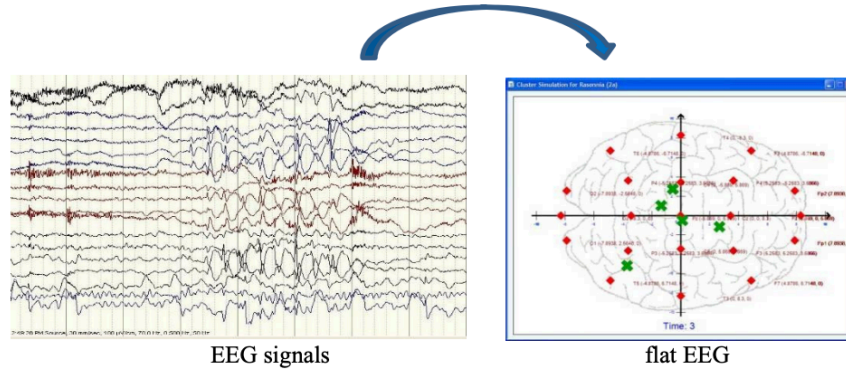


Fig. 1. Transformation of EEG signals into flat EEG

The EEG coordinate system is defined by a radius, denoted as  $r$ , which corresponds to the size of the patient's head (see Equation 1).

$$C = \left\{ \left( (u, v, w), e_{pot} \right) : u, v, w, e_{pot} \in \mathfrak{R} \text{ and } u^2 + v^2 + w^2 = r^2 \right\} \quad (1)$$

The mapping from the original EEG signal space to the two-dimensional plane is established as follows:  $c \ S \ C \ M \rightarrow$  such that

$$S \left( (u, v, w), e_{pot} \right) = \left( \frac{ru + irv}{r + w}, e_{pot} \right) = \left( \frac{ru}{r + w}, \frac{rv}{r + w} \right)_{e_{pot}(u, v, w)} \quad (2)$$

where  $M_c = \left\{ \left( (u, v)_0, e_{pot} \right) : u, v, e_{pot} \in \mathfrak{R} \right\}$  is the first component of Fuzzy Topographic Topological Mapping (FTTM).  $C$  and  $M_c$  were both 2-manifolds. Moreover,  $S$  is a one-to-one function and conformal. The transformation of fEEG into image was carried out by [9].

**Definition 1. Interval-Valued Fuzzy Set** [2][10]. Let  $I([0,1])$  be the set of all closed subintervals in  $[0,1]$  such that  $I([0,1]) = \left\{ \mathbf{x} = [x, \bar{x}] \mid (x, \bar{x}) \in [0,1]^2 \text{ and } x \leq \bar{x} \right\}$ . An interval-valued fuzzy set (IVFS), denoted  $A$  on the universe  $U \neq \emptyset$  is a mapping  $A: U \rightarrow I([0,1])$  such that the membership degree of  $u \in U$  is given by  $\mu_A(u) = [\mu_{\underline{A}}(u), \mu_{\bar{A}}(u)] \in I([0,1])$ , where  $\underline{A}: U \rightarrow [0,1]$  and  $\bar{A}: U \rightarrow [0,1]$  are mappings defining the lower and the upper bound of the membership interval respectively.

**Definition 2. Type-II Fuzzy Set** [11]. Let  $U$  be a nonempty set. A type-II fuzzy set, denoted  $\tilde{A}$ , is defined as an object of the form  $\tilde{A} = \left\{ (u, \mu_{\tilde{A}}(u)) \mid u \in U \right\}$ , where  $\mu_{\tilde{A}}(u)$  is a type-II membership function. The membership function lies in an interval range such that  $\mu_{upp} = [\mu_u(u)]^\alpha$  and  $\mu_{low} = [\mu_l(u)]^\alpha$ .

### 3. Methodology

In this section, a gray-scale image algorithm using the combination of IVFS and type-II is proposed. The step-bystep procedure for the proposed method is outlined as follows: Step 1 Define the interval-valued fuzzy set for contrast enhancement. Apply the lower bound and upper bound of the intensity interval. In this study, the lower and upper bound are set to 0.2 and 0.8, respectively.

$$hist(i) = \sum_x \sum_y Int(x, y) = i \quad (3)$$

Step 3 Normalize the histogram to obtain the cumulative distribution function (CDF). This encodes the fraction of pixels with an intensity that is equal to or less than a specific value.

$$C(i) = \frac{\sum_{j \leq i} h(j)}{N} \quad (4)$$

Step 4 Build the interval-valued fuzzy transformation function 4.1 Find the nearest indices in the CDF for the lower and upper bounds

Lower index = min (|cdf – lower bound|)

Upper index = min (|cdf – upper bound|)

4.2 Scale the pixel value to the range [0, 1]

$$Scaled = \frac{x - cdf_{lower\ index}}{cdf_{upper\ index} - cdf_{lower\ index}}$$

Step 5 Find the average of the scaled value 5.1 Calculate upper and lower membership degree

$$\mu_u(I_{ij}) = [\mu_{\tilde{A}}(I_{ij})]^\alpha \quad (5)$$

$$\mu_l(I_{ij}) = [\mu_{\tilde{A}}(I_{ij})]^\beta \quad (6)$$

5.2 Modify the membership function

$$\mu^{enh}(I_{ij}) = \mu_l(I_{ij}) \cdot \lambda + \mu_u(I_{ij}) \cdot (1 - \lambda), \quad 0 < \lambda < 1 \quad (7)$$

To perform comparisons and evaluate the quality of the reconstructed images, three metrics are employed: peaksignal-to-noise ratio (PSNR), mean square error (MSE), and structural similarity index measure (SSIM). The PSNR is a widely used metric that quantifies the ratio between the peak signal power and the noise power in an image. It is calculated as in Equation 8

$$PSNR = 10 \log_{10} \frac{MAX^2}{MSE} \quad (8)$$

where MAX represents the maximum possible pixel value and MSE is the mean square error between the original and reconstructed images. A higher PSNR value indicates a lower level of distortion or noise in the reconstructed image, thus signifying better quality. The MSE provides a measure of the average squared difference between corresponding pixels of the original and reconstructed images. It is calculated as (see Equation 9):

$$MSE = \frac{1}{N} \sum_{i=1}^N (I(i, j) - R(i, j))^2 \quad (9)$$

where  $I(i, j)$  and  $R(i, j)$  denote the pixel values at position  $(i, j)$  in the original and reconstructed images, respectively, and  $N$  represents the total number of pixels. A lower MSE value indicates less deviation between the original and reconstructed images, suggesting higher fidelity. The SSIM is a comprehensive metric that evaluates the structural similarity between the original and reconstructed images (see Equation 10). It incorporates assessments of luminance, contrast, and structural information. The SSIM is calculated based on local comparisons of pixel values and their statistics. A higher SSIM value indicates a greater similarity between the original and reconstructed images, implying better quality and preservation of structural details.

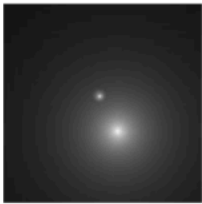
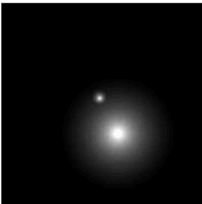
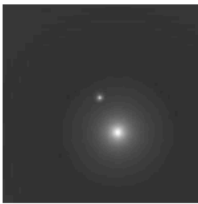
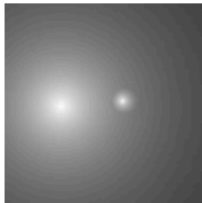
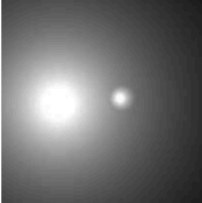
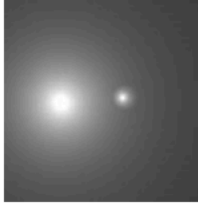
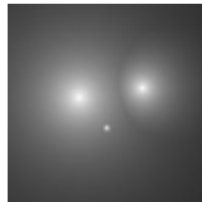


$$SSIM(L, M) = \frac{(2\mu_L\mu_M + c_1)(2\sigma_{LM} + c_2)}{(\mu_L^2 + \mu_M^2 + c_1)(\sigma_L^2 + \sigma_M^2 + c_2)} \quad (10)$$

where  $\mu_L$  and  $\mu_M$  represent the mean values of the input and output images, respectively. Meanwhile  $\sigma_L$  and  $\sigma_M$  represent their respective standard deviations. The standard deviations provide an estimation of contrast similarity between the images. The SSIM metric ranges between -1 and 1, where 1 indicates perfect similarity, 0 indicates no similarity, and -1 indicates perfect anti-correlation [12].

#### 4. Results and Discussion

In this section, the proposed method is applied to flat EEG input images corresponding to three different time points:  $t = 1, 2,$  and  $3$ . These images have a size of  $201 \times 201$  pixels. It can be observed that two clusters of electric current sources at  $t = 1$  and  $t = 2$ , while three clusters are detected at  $t = 3$ . The luminosity of the images represents the intensity of the electrical potential. Table 1 illustrates the output images obtained by implementing the proposed method and interval-valued method. The output images demonstrate that better results are achieved through the application of the proposed method. To evaluate the performance of the proposed method, we present a comparison in Table 2 between the input and output images for the flat EEG data. The peak-signal-to-noise ratio (PSNR), mean square error (MSE), and structural similarity index measure (SSIM) are carried out to perform the comparisons. From the values in Table 2, we can observe that the PSNR values are higher and MSE values are lower for the proposed method. The PSNR and MSE are metrics used to evaluate the quality of images. A higher PSNR value indicates that the reconstructed image is closer to the original image in quality. On the other hand, unlike the PSNR, lower value of MSE indicates better quality of images. The MSE calculates the average squared difference between corresponding pixel values. The SSIM values for the proposed method are higher and closer to 1 compared to the interval-valued approach. This indicates that the combination of interval-valued with type-II exhibits superior performance, as it demonstrates a higher degree of resemblance between the input and output images.

**Table 1**  
 The enhanced output of FEEG

Time (s)	Input Image	Interval-Valued	Proposed Method
$t = 1$			
$t = 2$			
$t = 3$			

**Table 2**  
 Performance comparisons

Time (s)	PSNR		MSE		SSIM	
	IVFS	Proposed	IVFS	Proposed	IVFS	Proposed
t = 1	16.5767	23.3672	1.4302e+03	299.4719	0.1805	0.9247
t = 2	9.6122	11.7932	7.1098e+03	4.3029e+03	0.9504	0.9747
t = 3	12.8573	14.7	3.3678e+03	2.2034e+03	0.7905	0.9750

## 5. Conclusions

This paper introduces an enhancement method for flat EEG images based on combination of interval-valued with type-II fuzzy sets. The method proposed in this study successfully addresses the issue of boundary fuzziness, resulting in improved boundary clarity while simultaneously preserving the crucial information related to cluster centers. In addition to qualitative assessments, the quantitative metrics of peak signal-to-noise ratio (PSNR), mean square error (MSE), and structural similarity index measure (SSIM) are utilized to objectively evaluate the performance and efficacy of the method. These metrics provide a numerical estimation of the errors and the degree of similarity between the input and output images, thereby offering supplementary measures to assess the effectiveness of the proposed method. The experimental outcomes demonstrate the superiority of the proposed method over the interval-valued fuzzy approach.

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